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Iterative Learning Control of Anti-Lock Braking System

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Abstract: **This paper analyzes the anti-lock braking system by iterative learning control. We usually use PID algorism, logic gate threshold value control and many other ways to simulate ABS, but the result is not ideal, so this paper use iterative learning control to design the ABS control algorithm, Simulation shows the algorism has a better effect and can be extended to multi-wheel system.**

Index Terms: *ABS*; *single-wheel*; *iterative learning control.*

1. Instruction

Automotive Anti-lock Braking System(ABS) is a device to automatically adjust the braking force, it can be said a typical optimal control system design problems.

When a car is in the process of moving in the event of an emergency situation, the driver usually slammed on the brake pedal to get the most desired braking effect, However, equipped with conventional brakes, its four wheels will soon be in a "lock" state, and namely has no scroll wheel, and the drag still slips on the road. This way will not bring the best braking effect, but bring negative effects.

Wheel sliding rates can be controlled within a certain range by ABS. We can take advantage of adhesion between tire and road, playing braking performance of brakes, improving braking deceleration and shortening braking distances. Eventually we get an effective improvement to the directional stability of the car, thus greatly improve the driving safety of the car.

Automobile braking process is a very complex activity, therefore the research for Auto anti-lock systems needs lots of environmental testing and laboratory testing platforms, which require a lot of money and time to get a similar data and test results. If we only consider the major factor, ignoring minor factors which affect the anti-lock system, we can use mathematical models to establish the actual process of anti-lock braking system abstract model. Mathematical modeling methods can not only save the time and funds, but also draw better relevant conclusions.

Arimmoto has proposed iterative learning algorithm, which is applied to many aspects of industry, Reference [1–6] introduce these working. Reference [7–13] use Matlab to simulate the ABS though ways of PID, fuzzy PID, and Logic gate threshold value control, etc.

2. Problem Description

The modeled of the vehicle dynamics model is a complex process. A vehicle dynamics model can be used in Newtonian physics vector mechanics system to establish each rigid body kinematic equations. Due to different the applications for different purposes, complexity extent of complexity of each model is different, but majority of them are based on classical mechanics, which is established by the calculation of the kinetic equations. Conventional vehicle dynamics performances include: running balance, movement stability and dynamic performance of the curve, At present there are generally single-wheel vehicle model, vehicle model two-wheel, four-wheel vehicle model and general vehicle model. Here the paper discussed only one wheel of the vehicle model.

During the design and analysis of control system based on the model, the wheel vehicle model was used, and its model shown in Fig. 1. u, r, ω denotes speed of the wheel center(wheel velocity), Wheel angular velocity and Wheel rolling radius respectively.

F_ω is air resistance, W is vehicle gravity, W=mg, where m is

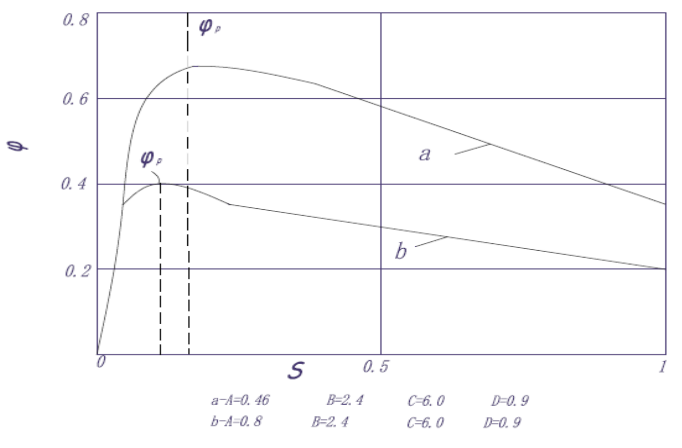

Fig. 1. $\varphi - \mathrm{s}$ simulation curves

vehicle quality; F_z is normal direction reaction force; T_b is braking torque; T_f is rolling resistance torque; F_b is ground braking force.

According to the knowledge of theoretical mechanics, the single-wheeled vehicle brake differential equation model can get.

$$\mathrm{m}\dot{u} = -\mathrm{F}_{\omega} - \mathrm{F}_b \tag{1}$$

$$\mathrm{J}\dot{\omega} = -\mathrm{F}_{br} - \mathrm{F}_b - \mathrm{T}_f \tag{2}$$

$$\mathrm{F}_b = \mathrm{F}_z \cdot \varphi \tag{3}$$

Where J is inertia moment of wheel. For drive wheel, in addition to calculating the rotational inertia moment J_q, it should also calculate the moment of inertia of the drive train J_T. So the single-wheeled model is $J = \frac{J_T}{2} + J_q$; φ is a vertical surface friction coefficient, it should be noted that φ is a function of the sliding rate.

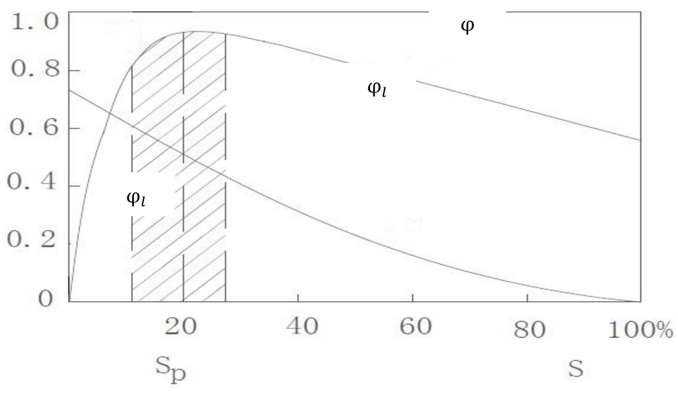

Fig. 2. $\varphi - \mathrm{s}\varphi_l - \mathrm{s}$ relationship curves

3. Iterative Learning Control

Although logic control algorithm has been widely used in automotive, it is not the best control algorithms. To further improve the performance of ABS, many papers presented control algorithms based on a sliding rate. If we use sliding rate as the control target, it is easy to achieve continuous control, Thereby we can enhance the ride performance of ABS in braking process, and maximize its braking performance. To achieve continuous braking, the simplest algorithm is PID control, which adjusts the proportion, integrability and derivation, these three coefficients properly. This paper use s iterative learning control method on the basis of the traditional PID algorithm to achieve better control. Set slip rate is s for setting goals, the control error is:

$$\mathrm{e}_k(t) = s(t) - s_0(t) \tag{4}$$

So traditional PID control law can be expressed as Eq. (5).

$$u_k(t) = k_p \mathrm{e}_k(\mathrm{t}) + k_i \int_0^t \mathrm{e}_k(\mathrm{t})\mathrm{d}t + k_d \frac{\mathrm{de}}{\mathrm{d}t} \tag{5}$$

Consider iterative PID type learning law

$$u_{k+1}(\mathrm{t}) = \mathrm{u}_k(\mathrm{t}) + \Gamma(\dot{e_k}(\mathrm{t}) + \mathrm{Q}_0 \mathrm{e}_k(\mathrm{t})) + \mathrm{Q}_1 \int_0^t \mathrm{e}_k(\tau)\mathrm{d}\tau \tag{5a}$$

Unified written Eq. (6)

$$u_{k+1}(\mathrm{t}) = \mathrm{u}_k(\mathrm{t}) + \Gamma(\dot{e_k}(\mathrm{t}) + \mathrm{P}(\mathrm{e}_k(\mathrm{t}))) \ \ t \in [0, T] \tag{6}$$

Where P is error function.

Lemma 1. let metrics is canonical for (E, A), $\widehat{\mathrm{E}} = (\lambda\mathrm{E} - \mathrm{A})^{-1} E$, $ind(E) = v$. For the ABS system, the desired trajectory $\mathrm{s(t)}$ is given and can be reached, if meet $(\mathrm{H}_3)\widehat{\mathrm{B}} \in \mathrm{R}(\hat{E}^v)$, here $\widehat{\mathrm{B}} = (\lambda\mathrm{E} - \mathrm{A})^{-1} B$;

$(\mathrm{H}_4)\ I - \Gamma C \hat{E}^D \hat{B} \leqslant \rho < 1$;

$(\mathrm{H}_5)\ x^k(0) = \hat{E}^D \hat{E} x^0 = x^0 \neq x^d(0),\ k = 0, 1, 2, \ldots.$

Then the ABS system is under the effect of the learning law (6).

$$\lim_{k \to \infty} s_0(t) = s(t) + \tilde{e}(t)\,(t \in [0, T])$$

Where $\tilde{e}(t)$ is canonical of the system (7).

$$\begin{cases} \dot{\tilde{e}}(t) + P[\tilde{e}(t)] = 0 \\ \tilde{e}(0) = C[x_d(0) - x_0] \end{cases} \tag{7}$$

Proof. According to lemma1, under condition (H_3), the ABS system set with all compatible initial value is $\mathrm{x} \in R^n$; $[\mathrm{I} - \hat{E}^D \widehat{\mathrm{E}}]\mathrm{x} = 0$,all controls $\mathrm{u(t)} \in \mathrm{C(I)}$ about the compatibility initial value are permitted. Taking control input $\mathrm{u^*(t)}$ makes system initial state situated x_0 control output is $\mathrm{y(t)}$. According to Lemma 1, we know

$$\begin{cases} \mathrm{x}(\mathrm{t}) = e^{E^D A(t_0)} \hat{E}^D \widehat{\mathrm{E}} x^0 + \int_0^t e^{\hat{E}^D \hat{A}(t-s)} \hat{E}^D \hat{B} u(s)\mathrm{d}s \\ y(t) = Cx(t) = y_d(t) - \tilde{e}(t) \end{cases} \tag{8}$$

Let $\Delta \mathrm{x}_k(t)=x(t)-\mathrm{x}_k(t)$, $\Delta \mathrm{u}_k(t)=u(t)-\mathrm{u}_k(t)$, so

$$\begin{cases}\Delta \dot{\mathrm{x}}_k(t)=\hat{E}^D\hat{A}\Delta \mathrm{x}_t(t)+\hat{E}^D\hat{B}\Delta u_k(t)\\ \Delta \mathrm{x}_k(0)=0\end{cases} \tag{9}$$

By the equation (6) and Equation (8), $k+1$ times iteration control error written in

$$\begin{aligned}\Delta \mathrm{u}_{k+1}(t)&=\Delta \mathrm{u}_k(t)-\Gamma\left[\dot{e_k}(\mathrm{t})+\mathrm{P}\left(\mathrm{e}_k(\mathrm{t})\right)\right]\\&=\Delta \mathrm{u}_k(t)-\Gamma\left[\mathrm{C}\Delta \dot{x}_k(\mathrm{t})-\mathrm{P}\left(\tilde{e}(t)\right)+\right.\\&\quad \left.P\left(C\Delta x_k(t)+\tilde{e}_k(t)\right)\right]\\&=\left[I-\Gamma \mathrm{C}\hat{E}^D\hat{B}\right]\Delta \mathrm{u}_k(t)-\Gamma \mathrm{C}\hat{E}^D\hat{A}\Delta \mathrm{x}_k(t)-\\&\quad \Gamma \mathrm{P}(\mathrm{C}\Delta \mathrm{x}_k(t))\end{aligned} \tag{10}$$

As P is a linear operator, so there is a normal number b_0 that makes

$$\|\mathrm{P}(\mathrm{C}\Delta \mathrm{x}_k(t))\|\leqslant \mathrm{b}_0\|\Delta \mathrm{x}_k(t)\| \tag{11}$$

Take equation (10) α norm at both ends.

$$\begin{aligned}\|\Delta \mathrm{u}_{k+1}(t)\|_\alpha \leqslant \left\|I-\Gamma \mathrm{C}\hat{E}^D\hat{B}\right\|\|\Delta \mathrm{u}_k(t)\|_\alpha +\\ \mathrm{b}_1\|\Delta \mathrm{x}_k(t)\|_\alpha,\end{aligned} \tag{12}$$

Where $\mathrm{b}_1=\left\|\Gamma \mathrm{C}\hat{E}^D\hat{A}\right\|+\mathrm{b}_0$, by equation (9)

$$\|\Delta \mathrm{x}_k(t)\|_\alpha \leqslant \int_0^{\mathrm{t}}\left\|\hat{E}^D\hat{B}\right\|e^{a(t-s)}\|\Delta \mathrm{u}_k(s)\|\,\mathrm{d}s \tag{13}$$

Equation (13) multiplied by the positive ends of the same function $e^{at}(t\in[0,\mathrm{T}],\ \alpha>0)$ can get

$$\|\Delta \mathrm{x}_k(t)\|_\alpha \leqslant \left\|\hat{E}^D\hat{B}\right\|\frac{1-e^{-(\alpha-a)^T}}{\alpha-a}\|\Delta \mathrm{u}_k(t)\|_\alpha \tag{14}$$

Let equation (14) into equation (12) and get equation (15)

$$\|\Delta \mathrm{x}_{k+1}(t)\|_\alpha \leqslant \left[\|\rho+\mathrm{b}_1\|\,\hat{E}^D\hat{B}\frac{1-e^{-(\alpha-a)^T}}{\alpha-a}\right]\|\Delta \mathrm{u}_k(t)\|_\alpha \tag{15}$$

As $0<\rho<1$, taking $\alpha>0$ large enough so that

$$\rho+\mathrm{b}_1\ \hat{E}^D\hat{B}\ \frac{1-e^{-(\alpha-a)^T}}{\alpha-a}<1 \tag{16}$$

By equation(15),we get

$$\lim_{\mathrm{k}\to\infty}\|\Delta \mathrm{u}_k(t)\|_\alpha=0 \tag{17}$$

By equation (14) and equation (17), we know $\lim_{\mathrm{k}\to\infty}\|\Delta \mathrm{x}_k\|_\alpha=0$,as $\sup_{t\in[0,T]}\|\Delta \mathrm{x}_k\|\leqslant e^{\alpha T}\|\Delta \mathrm{x}_k\|_\alpha$, So $\lim_{k\to\infty}\sup_{t\in[0,T]}\|\Delta \mathrm{x}_k\|=0$, $\lim_{k\to\infty}\mathrm{s}_0(t)=^*_s(t)=s(t)-C\tilde{e}(t)$ QED.

When $P\left(e_k(t)\right)=Q_0e_k(t)$system (7) is

$$\begin{cases}\dot{\tilde{e}}(t)+Q_0\left[\tilde{e}(t)\right]=0\\ \tilde{e}(0)=C[x_d(0)-x_0]\end{cases} \tag{18}$$

Solution of equation (18) can be obtained $\tilde{e}(t)=\mathrm{e}^{-Q_0t}C(x_d(0)-x_0)$.

Corollary 1. let matrix is canonical for (E, A), $\widehat{\mathrm{E}}=(\lambda \mathrm{E}-\mathrm{A})^{-1}E$, $ind(E)=v$, for the ABS system given desired trajectory $\mathrm{s(t)}(\mathrm{t}\in[0,\mathrm{T}])$, and meet the following condition $(H_6)\ \hat{B}\in R\left(\hat{E^v}\right)$, where $\hat{B}=(\lambda E-A)^{-1}B$;

$(H_7)\ I-\Gamma C\hat{E}^D\hat{B}\leqslant\rho<1$

$(\mathrm{H}_8)\ x^k(0)=\hat{E}^D\hat{E}x^0=x^0\neq x^d(0),\ k=0,1,2,\ldots.$

Then the ABS system under effect of PID learning law (5a)

$$\lim_{k\to\infty}s(t)=s_0(t)+C^*e^{-A^*t}R^*C\left(x_d(0)-x_0\right),\ t\in[0,T]$$

Note: learning law (6) is a class very wide range of learning rule; we select the appropriate linear operator P to effectively control the state of the early offset. For example, take the following operator

i) $\mathrm{P}\left(\mathrm{e}\left(\mathrm{t}\right)\right)=-\mathrm{tRe}(t)$

ii) $\mathrm{P}\left(\mathrm{e}\left(\mathrm{t}\right)\right)=-\Phi(\mathrm{t})\mathrm{Re(t)}$

iii) $\mathrm{P}\left(\mathrm{e}\left(\mathrm{t}\right)\right)=-\mathrm{Q(t)Re(t)}$

Here Φ (t) is a real-valued function $(\Phi.[0,T]\to R)$,Q(t) is defined in range of $[0,T]$ that is a matrix-valued function, and satisfies

$$\sup_{t\in[0,T]}|\Phi(t)|\leqslant b_{\Phi<\infty},\quad \sup_{t\in[0,T]}|\mathrm{Q}(t)|\leqslant b_{\mathrm{Q}}<\infty,$$

Theorem 1 corresponds to the conclusion, there is

i) $\lim_{k\to\infty}s(t)=s_0(t)+\mathrm{e}^{\frac{1}{2}Rt^2}C\left(x_d(0)-x_0\right)\ t\in[0,T]$;

ii) $\lim_{k\to\infty}s(t)=s_0(t)+\mathrm{e}^{\int_0^t\Phi(\tau)\mathrm{d}\tau}C\left(x_d(0)-x_0\right)\ t\in[0,T]$;

iii) $\lim_{k\to\infty}s(t)=s_0(t)+\mathrm{e}^{\int_0^tQ(\tau)d\tau}C\left(x_d(0)-x_0\right)\ t\in[0,T]$.

4. Conclusions

Form Fig. 1, $\varphi-\mathrm{s}$ curve in s_p is divided into two areas, the left side of the s_p is stable braking region, and the right of s_p is non-stable braking region. By calculation we find that when the target s_0 is set on the left s_p, through iterative learning controller can make the wheel slip ratio rapidly approaching s_0, and the braking approximate process ideally. If take s_0 to set s_p point, then we can form the stable limit cycle as sliding rate s_p. Especially s_0 set slightly to the right of s_p, sliding rate is relatively fluctuations. In practice, peak adhesion coefficient has large change (between

5%–30%). Once the s_0 is set the right of s_p, the sliding rate has large fluctuation, For example ,when setting s_0 on the farther right of s_p, the wheels have a risk of locking. If we use conservative election s_0, although it guaranteed s_0 on the left s_p, and solve the problem of non-stability system, however in most a lot of road conditions, road surface friction coefficient will not be fully utilized. Thereby the wheel lose slip rate control algorithm superiority. Form the dynamic adjustment process of iterative learning control we get that using the slip rate as the control target can solve as following: the real-time identify changes of the road surface adhesion coefficient; automatically change the control objectives s_0 to track changes of the road surface adhesion coefficient, so that brake can always be in the best condition. This shows that the use of iterative learning control algorithm as the ABS controller can satisfy requirements in all conditions, with the online features tuning controller parameters.

Acknowledgements

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